Introduction to Dynare and local approximation

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Why Dynare?

- Dynare (mainly) deals with structural models that rest on theory
- Microeconomic foundations $\Rightarrow$ nonlinear models
- Intertemporal optimization $\Rightarrow$ expectations matter $\Rightarrow$ Perfect foresight or Rational expectations
- (Stochastic) Shocks push the economy away from equilibrium and endogenous dynamics bring it back towards equilibrium
- Solving nonlinear (stochastic) forward-looking model

Dynare

- A matlab/octave toolbox with algorithms handling DSGE models
- A modeling language to represent models and computing tasks
- A clear separation between:
  - The model declared by the user → the preprocessor
  - The computing functions → a collection of matlab routines.

Dynare can...

- Compute the steady state of a model
- Solve a perfect foresight model
- Solve local approximations of stochastic models
- Estimate a model (full or limited information)
- Check for identification of the parameters
- Compute optimal policy
- Perform global sensitivity analysis

...

A simple model

$$\frac{1}{c_t} = \beta \mathbb{E}_t \left[\frac{\alpha e^{a_{t+1}} k_{t+1}^{\alpha-1} + 1 - \delta}{c_{t+1}} \right] \tag{1}$$

$$k_{t+1} = e^{a_t} k_t^{\alpha} + (1-\delta) k_t - c_t \tag{2}$$

$$a_t = \varphi a_{t-1} + \epsilon_t \tag{3}$$

- $\{\epsilon_t\} \overset{iid}{\sim} \mathcal{N}(0, \sigma_\epsilon^2)$
- $\mathbb{E}_t[X_{t+1}]$ is the expectation conditional on the information available at time t.
- At time t all the realisations of the variables (from time ∞ to t) are known, the model is known.

Log linearization

- Suppose that we have the following recurrent equation:

$$x_t = f(x_{t-1})$$

  with the steady state $x^\star$ such that $x^\star = f(x^\star) \neq 0$.

- Define $\tilde{x}_t$ such that $x_t = x^\star e^{\tilde{x}_t}$, the percentage deviation from the steady state. We have:

$$x^\star e^{\tilde{x}_t} = f\left(x^\star e^{\tilde{x}_{t-1}}\right)$$

- A first order Taylor approximation of both sides around $\tilde{x}_t = 0$ gives:

$$x^\star + x^\star \tilde{x}_t \approx f(x^\star) + x^\star f'(x^\star)\tilde{x}_{t-1}$$

$$\Leftrightarrow \tilde{x}_t \approx f'(x^\star)\tilde{x}_{t-1}$$

Linearized simple model

- TFP is already log-linearized
- We can show that the first order approximations of the Euler equation and physical capital law of motion are:

$$\mathbb{E}_t\left[\tilde{c}_t - \tilde{c}_{t+1} + \frac{\rho+\delta}{1+\rho}\Big(a_{t+1} - (1-\alpha)\tilde{k}_{t+1}\Big)\right] = 0 \tag{4}$$

$$\tilde{k}_{t+1} = \frac{y^\star}{k^\star}a_t + \beta^{-1}\tilde{k}_t - \frac{c^\star}{k^\star}\tilde{c}_t \tag{5}$$

- The deterministic steady state must satisfy the follwing system of equations:

$$\begin{cases} a^\star &= \varphi a^\star \\ 1 &= \beta\left(\alpha\ k^{\star\ \alpha-1} + 1 - \delta\right) \\ y^\star &=\ k^{\star\ \alpha} \\ c^\star &= y^\star - \delta k^\star \end{cases}$$

for which we easily obtain a closed form solution:

$$\begin{cases} a^\star &= 0 \\ k^\star &= \left(\frac{\alpha}{\rho+\delta}\right)^{\frac{1}{1-\alpha}} \\ y^\star &=\ k^{\star\ \alpha} \\ c^\star &= y^\star - \delta k^\star \end{cases}$$

with $\beta^{-1} = 1 + \rho$.

- **Log-linearization of the ressource constraint.** We can rewrite (2) equivalently as:

$$k^\star e^{\tilde{k}_{t+1}} = e^{a_t} k^\star e^{\tilde{k}_t} + (1-\delta) k^\star e^{\tilde{k}_t} - c^\star e^{\tilde{c}_t}$$

With a first order Taylor approximation (around $\tilde{k} = \tilde{c} = a = 0$) on both sides (omitting the cross products since we are looking for a first order approximation):

$$k^\star + k^\star \tilde{k}_{t+1} \approx\ k^{\star\ \alpha} + a_t\ k^{\star\ \alpha} + \alpha\ k^{\star\ \alpha-1}\,\tilde{k}_t + (1-\delta)\left(k^\star + k^\star \tilde{k}_t\right) - c^\star - c^\star \tilde{c}_t$$

Removing $k^\star = (1-\delta)k^\star +\ k^{\star\ \alpha} - c^\star$ on both sides:

$$k^\star \tilde{k}_{t+1} \approx a_t\ k^{\star\ \alpha} + \alpha\ k^{\star\ \alpha-1}\,\tilde{k}_t + (1-\delta) k^\star \tilde{k}_t - c^\star \tilde{c}_t$$

Dividing by $k^\star$ on both sides and exploiting steady state restrictions (definition of $y^\star$ and Euler equation at the steady state), we finally obtain:

$$\tilde{k}_{t+1} \approx \frac{y^\star}{k^\star} a_t + \beta^{-1} \tilde{k}_t - \frac{c^\star}{k^\star} \tilde{c}_t$$

- **Log-linearization of the Euler equation.** We can rewrite (1) equivalently as:

$$1 = \beta \mathbb{E}_t \left[e^{\tilde{c}_t - \tilde{c}_{t+1}} \left(\alpha e^{a_t} \left(k^\star e^{\tilde{k}_t} \right)^{\alpha - 1} + 1 - \delta \right) \right]$$

With a first order Taylor expansion under the conditional expectation:

$$1 \approx \beta \left(\alpha \ k^{\star \ \alpha-1} + 1 - \delta \right) + \beta \left(\alpha \ k^{\star \ \alpha-1} + 1 - \delta \right) \left(\tilde{c}_t - \tilde{c}_{t+1} \right) + \beta \alpha \ k^{\star \ \alpha-1} \left(a_{t+1} - (1-\alpha) \tilde{k}_{t+1} \right)$$

Since the steady state satisfies $\beta \left(\alpha \ k^{\star \ \alpha-1} + 1 - \delta \right) = 1$ and $\alpha \ k^{\star \ \alpha-1} = \rho + \delta$, we have:

$$0 \approx \tilde{c}_t - \tilde{c}_{t+1} + \frac{\rho + \delta}{1 + \rho} \left(a_{t+1} - (1-\alpha) \tilde{k}_{t+1} \right)$$

Exact solution of the linearized simple model, I

- A solution is an invariant mapping between the controls and the states
- If $c_t = \psi(k_t, a_t)$ is known, one can build time series for all the endogenous variables by iterating over (2)-(3)
- Since the model is (log) linearized, we postulate a linear solution:

$$\begin{aligned} \tilde{c}_t &= \eta_{ck}\tilde{k}_t + \eta_{ca}a_t \\ \tilde{k}_{t+1} &= \eta_{kk}\tilde{k}_t + \eta_{ka}a_t \end{aligned} \tag{6}$$

- A unique (stable) solution exists iff there exists a unique vector $(\eta_{ck}, \eta_{ca}, \eta_{kk}, \eta_{ka})$ such that (6) is consistent with (4), (5) and (3)

Exact solution of the linearized simple model, II

- Substituting (6) in (4), (5) and (3), one can show that the reduced form parameters must satisfy:

$$\begin{cases} \eta_{ck} & = \frac{k^\star}{c^\star}\left(\beta^{-1} - \eta_{kk}\right) \\ \eta_{ca} & = \frac{y^\star}{c^\star} - \frac{k^\star}{c^\star}\eta_{ka} \\ 0 & = \frac{k^\star}{c^\star}\left(\beta^{-1} - \eta_{kk}\right)(1 - \eta_{kk}) - (1-\alpha)\frac{\rho+\delta}{1+\rho}\eta_{kk} \\ 0 & = \left(\frac{y^\star}{c^\star} - \frac{k^\star}{c^\star}\eta_{ka}\right)(1-\varphi) - \frac{k^\star}{c^\star}\left(\beta^{-1} - \eta_{kk}\right)\eta_{ka} + \frac{\rho+\delta}{1+\rho}\left(\varphi - (1-\alpha)\eta_{ka}\right) \end{cases}$$

- The third equation, for η_{kk} is quadratic

- If we can identify a unique stable solution for η_{kk}, we can deduce η_{ca}, η_{ca} and η_{ck}

Exact solution of the linearized simple model, III

- η_{kk} must solve:

$$\eta_{kk}^2 - \xi\eta_{kk} + \beta^{-1} = 0$$

  where:

$$\xi = 1 + \beta^{-1} + \frac{c^\star}{k^\star}(1-\alpha)\frac{\rho+\delta}{1+\rho} > 1 + \beta^{-1}$$

- The equation admits two real roots one unstable (greater than 1) one stable (between 0 and 1)

- The stable solution is:

$$\eta_{kk} = \frac{\xi}{2} - \sqrt{\left(\frac{\xi}{2}\right)^2 - \beta^{-1}}$$

- When solving (linearized) RE model we always have to deal with a quadratic equations and to rule out unstable solutions

Exact solution of the linearized simple model, IV

- The endogenous variables are ARMA processes
- For instance, the output is characterized by:

$$\begin{cases} \tilde{y}_t &= a_t + \alpha \tilde{k}_t \\ \tilde{k}_t &= \eta_{kk} \tilde{k}_{t-1} + \eta_{ka} a_{t-1} \\ a_t &= \varphi a_{t-1} + \epsilon_t \end{cases}$$

$$\Rightarrow \tilde{y}_t = (\eta_{kk} + \varphi)\tilde{y}_{t-1} - \eta_{kk}\varphi\tilde{y}_{t-2} + \epsilon_t - (\eta_{kk} - \alpha\eta_{ka})\epsilon_{t-1}$$

an ARMA(2,1) stochastic process with two real roots in the autoregressive part (η_{kk} and φ).

Dynare code for the simple model, I

```
var c k y a;

varexo e;

parameters beta alpha delta phi;

beta = 0.98;
alpha = 0.33;
delta = 0.02;
phi = 0.98;

model;

  1/c = beta*((alpha*exp(a(1))*k^(alpha-1)+1-delta)/c(1));

  y = exp(a)*k(-1)^alpha;

  k = (1-delta)*k(-1) + y - c;

  a = phi*a(-1) + e;

end;
```

Dynare code for the simple model, II

```
steady_state_model;

  rho = 1/beta-1;

  a = 0;

  k = (alpha/(rho+delta))^(1/(1-alpha));

  y = k^alpha;

  c = y - delta*k;

end;

steady;
```

```
STEADY-STATE RESULTS:

c    2.35379
k    22.9753
y    2.8133
a    0
```

Dynare code for the simple model, III

```
stoch_simul(order=1, irf=200) a c k y;
```

```
MODEL SUMMARY

  Number of variables:         4
  Number of stochastic shocks: 1
  Number of state variables:   2
  Number of jumpers:           2
  Number of static variables:  1


MATRIX OF COVARIANCE OF EXOGENOUS SHOCKS
Variables           e
e            0.000100
```

Dynare code for the simple model, IV

POLICY AND TRANSITION FUNCTIONS

	a	c	k	y
Constant	0	2.353795	22.975287	2.813300
k(-1)	0	0.062248	0.958160	0.040408
a(-1)	0.980000	1.054477	1.702557	2.757034
e	1.000000	1.075997	1.737304	2.813300

THEORETICAL MOMENTS

VARIABLE	MEAN	STD. DEV.	VARIANCE
a	0.0000	0.0503	0.0025
c	2.3538	0.1543	0.0238
k	22.9753	1.7196	2.9569
y	2.8133	0.2021	0.0408

MATRIX OF CORRELATIONS

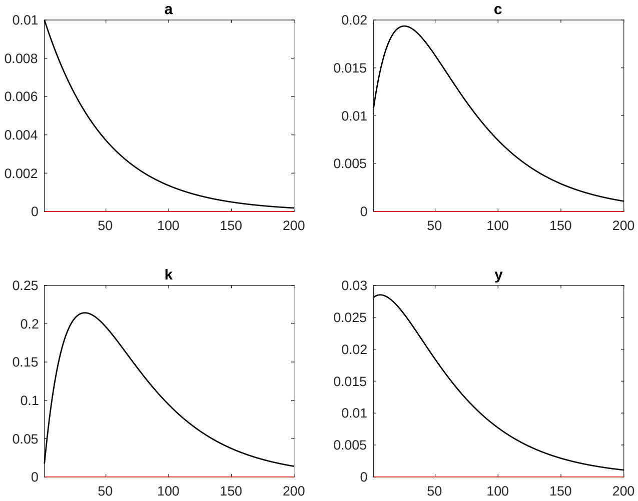
Variables	a	c	k	y
a	1.0000	0.9160	0.8323	0.9800
c	0.9160	1.0000	0.9848	0.9775
k	0.8323	0.9848	1.0000	0.9259
y	0.9800	0.9775	0.9259	1.0000

COEFFICIENTS OF AUTOCORRELATION

Order	1	2	3	4	5
a	0.9800	0.9604	0.9412	0.9224	0.9039
c	0.9974	0.9942	0.9903	0.9858	0.9808
k	0.9996	0.9983	0.9963	0.9936	0.9902
y	0.9902	0.9802	0.9700	0.9597	0.9491

Total computing time : 0h00m01s

Dynare code for the simple model, V



Remarks

- By default (with `order=1`) Dynare performs a linearization
- Use option `loglinear` to log-linearize the model
- If some variables are zero at the steady state:
  - Use the reparameterization explained above
  - Add reporting variables in the model:

```
logX = 100*log(X/STEADY_STATE(X));
```

General problem

- Let y be a $n \times 1$ vector of endogenous variables, u is a $q \times 1$ vector of innovations (exogenous variables in dynare language)
- We consider the following type of model:

$$\mathbb{E}_t \left[f(y_{t+1}, y_t, y_{t-1}, u_t) \right] = 0$$

with:

$$\begin{aligned} u_t &= \sigma \epsilon_t \\ \mathbb{E}[\epsilon_t] &= 0 \\ \mathbb{E}[\epsilon_t \epsilon_t'] &= \Sigma_\epsilon \end{aligned}$$

where σ is a scale parameter, ϵ is a vector of auxiliary random variables

- **Assumption** $f: \quad \mathbb{R}^{3n+q} \rightarrow \mathbb{R}^n$ is a differentiable function in $\mathcal{C}^k$

The solution

- We are looking for time invariant policy rules and transition equations:

$$y_t = g(y_{t-1}, u_t, \sigma)$$

$$\begin{aligned} \Rightarrow y_{t+1} &= g(y_t, u_{t+1}, \sigma) \\ &= g(g(y_{t-1}, u_t, \sigma), u_{t+1}, \sigma) \end{aligned}$$

- Define:

$$F_g(y_{t-1}, u_t, u_{t+1}, \sigma) = f(g(g(y_{t-1}, u_t, \sigma), u_{t+1}, \sigma), g(y_{t-1}, u_t, \sigma), y_{t-1}, u_t)$$

- Our problem can be restated as:

$$\mathbb{E}_t \left[F_g(y_{t-1}, u_t, u_{t+1}, \sigma) \right] = 0$$

- Solve a DSGE model $\Leftrightarrow$ Identify the unknown function g

Steady state

- A deterministic steady state, $y^\star$, for the model satisfies

$$f(y^\star, y^\star, y^\star, 0) = 0$$

- A model can have several steady states, but only one of them will be used for approximation.

- Furthermore, the solution function satisfies:

$$y^\star = g(y^\star, 0, 0)$$

- If the analytical steady state is available, it should be provided to dynare.

First order approximation, I

- Let $\hat{y} = y_{t-1} - y^\star$, $u = u_t$, $u_+ = u_{t+1}$, $f_{y_+} = \frac{\partial f}{\partial y_{t+1}}$, $f_y = \frac{\partial f}{\partial y_t}$, $f_{y_-} = \frac{\partial f}{\partial y_{t-1}}$, $f_u = \frac{\partial f}{\partial u_t}$, $g_y = \frac{\partial g}{\partial y_{t-1}}$, $g_u = \frac{\partial g}{\partial u_t}$, $g_\sigma = \frac{\partial g}{\partial \sigma}$. All the derivatives are evaluated at the deterministic steady state.

- With a first order Taylor expansion of F around $y^\star$:

$$
\begin{aligned}
0 \simeq F_g^{(1)}(y_-, u, u_+, \sigma) = \\
f_{y_+}\left(g_y\left(g_y \hat{y} + g_u u + g_\sigma \sigma\right) + g_u u_+ + g_\sigma \sigma\right) \\
+ f_y\left(g_y \hat{y} + g_u u + g_\sigma \sigma\right) + f_{y_-}\hat{y} + f_u u
\end{aligned}
$$

- **What has changed?** We now have three unknown "parameters" (g_y, g_u and g_σ) instead of an infinite number of parameters (function g).

First order approximation, II

- Taking the conditional expectation conditional and factorizing:

$$0 \simeq \left(f_{y_+} g_y g_y + f_y g_y + f_{y_-}\right)\hat{y} + \left(f_{y_+} g_y g_u + f_y g_u + f_u\right)u$$
$$+ \left(f_{y_+} g_y g_\sigma + f_{y_+} g_\sigma + f_y g_\sigma\right)\sigma$$

- This "equality" must hold for any value of $(\hat{y}, u, \sigma) \Rightarrow$ the terms between brackets must be zero. We have three (multivariate) equations and three (multivariate) unknowns:

$$\begin{cases} 0 &= f_{y_+} g_y g_y + f_y g_y + f_{y_-} \\ 0 &= f_{y_+} g_y g_u + f_y g_u + f_u \\ 0 &= f_{y_+} g_y g_\sigma + f_{y_+} g_\sigma + f_y g_\sigma \end{cases}$$

First order approximation, III

Certainty equivalence

- Assuming that g_y is known, we must have:

$$f_{y_+} g_y g_\sigma + f_{y_+} g_\sigma + f_y g_\sigma = 0$$

- Solving for g_σ, we obtain:

$$g_\sigma = 0$$

- This is a manifestation of the certainty equivalence property of the first order approximation: the policy rules and transition equations do not depend on the size of the structural shocks.

- In this sense, *future uncertainty* does not matter.

First order approximation, IV

Recovering the marginal effect of contemporaneous innovations, g_u

- Assuming that g_y is known, we must have:

$$f_{y_+} g_y g_u + f_y g_u + f_u = 0$$

- Solving for g_u, we obtain

$$g_u = -\left(f_{y_+} g_y + f_y\right)^{-1} f_u$$

- Note that $f_{y_+} g_y + f_y$ must be a full rank matrix

- g_u is the marginal effect of the structural innovations on the endogenous (jumping and states) variables

- Future uncertainty does not matter, but the *contemporaneous* innovations do affect the endogenous variables

First order approximation, V

Recovering the marginal effect of the past, g_y

- We must have:

$$\left(f_{y_+} g_y g_y + f_y g_y + f_{y_-}\right) \hat{y} = 0 \quad \forall \hat{y}$$

- This is a quadratic equation, but the unknown is a matrix! It is generally impossible to solve this equation analytically as we would do for a univariate quadratic equation

- If we interpret g_y as a lead operator, we can rewrite the equation as a second order recurrent equation:

$$f_{y_+} \hat{y}_{t+1} + f_y \hat{y}_t + f_{y_-} \hat{y}_{t-1} = 0$$

- For a given initial condition, $\hat{y}_{t-1}$, we have many paths $(\hat{y}_t, \hat{y}_{t+1})$ consistent with the second order recurrent equation

- We need another condition to pin-down a unique solution.

First order approximation, VI

Recovering the marginal effect of the past, g_y

- We can rewrite the second order recurrent equation as a first order recurrent equation for $z_t \equiv (\hat{y}'_t, \hat{y}'_{t+1})'$:

$$\begin{pmatrix} 0_n & f_{y+} \\ I_n & 0_n \end{pmatrix} \underbrace{\begin{pmatrix} \hat{y}_t \\ \hat{y}_{t+1} \end{pmatrix}}_{z_t} = \begin{pmatrix} -f_{y-} & -f_y \\ 0_n & I_n \end{pmatrix} \underbrace{\begin{pmatrix} \hat{y}_{t-1} \\ \hat{y}_t \end{pmatrix}}_{z_{t-1}}$$

- An admissible path z_t must also be such that the transitions, from $t-1$ to t or from t to $t+1$, are time invariant: *ceteris paribus* we have $\hat{y}_t = g_y \hat{y}_{t-1}$ **and** $\hat{y}_{t+1} = g_y \hat{y}_t$.

- In the sequel we examine the conditions under which g_y exists and allows to pin down **a unique stable trajectory** for the endogenous variables.

First order approximation, VII

Recovering the marginal effect of the past, g_y

$$\underbrace{\begin{pmatrix} 0_n & f_{y+} \\ I_n & 0_n \end{pmatrix}}_{D} \underbrace{\begin{pmatrix} I_n \\ g_y \end{pmatrix} g_y \hat{y}}_{z_t} = \underbrace{\begin{pmatrix} -f_{y-} & -f_y \\ 0_n & I_n \end{pmatrix}}_{E} \underbrace{\begin{pmatrix} I_n \\ g_y \end{pmatrix} \hat{y}}_{z_{t-1}}$$

$$\Leftrightarrow Dz_t = Ez_{t-1}$$

- Stability of the dynamical system is related to the eigenvalues (a stable eigenvalue is smaller than one in modulus)
- But matrix D is not necessarily invertible
- We use a generalized Schur decomposition of matrices D and E and compute generalized eigenvalues

First order approximation, VIII

- The real generalized Schur decomposition of the pencil $< E, D >$:

$$D = QTZ$$
$$E = QSZ$$

  with T upper triangular, S quasi-upper triangular, $Q'Q = I$ and $Z'Z = I$

- Generalized eigenvalues λ_i solves

$$\lambda_i D v_i = E v_i$$

  For diagonal blocks on S of dimension 1×1:

  - $T_{ii} \neq 0$: $\lambda_i = \frac{S_{ii}}{T_{ii}} \in \mathbb{R}$
  - $T_{ii} = 0$, $S_{ii} > 0$: $\lambda = +\infty$
  - $T_{ii} = 0$, $S_{ii} < 0$: $\lambda = -\infty$
  - $T_{ii} = 0$, $S_{ii} = 0$: $\lambda \in \mathbb{C}$

  Diagonal blocks of dimension 2×2 correspond to conjugate complex eigenvalues.

First order approximation, IX

Recovering the marginal effect of the past, g_y

- Applying the Schur decomposition and multiplying by Q' we obtain:

$$\begin{pmatrix} T_{11} & T_{12} \\ 0 & T_{22} \end{pmatrix} \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} I_n \\ g_y \end{pmatrix} g_y \hat{y} = \begin{pmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{pmatrix} \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} I_n \\ g_y \end{pmatrix} \hat{y}$$

- Matrices S and T are arranged in such a way that the stable eigenvalues come first.
- First block of lines, in S and T are for the stable eigenvalues. The rows of Z are partitioned accordingly.
- The columns of Z are partitioned consistently with I_n and g_y.

First order approximation, X

Recovering the marginal effect of the past, g_y

- To exclude explosive trajectories, one must impose

$$Z_{21} + Z_{22} g_y = 0$$

- Or equivalently:

$$g_y = -Z_{22}^{-1} Z_{21}$$

- A unique stable trajectory exists if Z_{22} is square **and** non-singular.

Blanchard and Kahn's condition

A unique stable trajectory exists if there are as many roots larger than one in modulus as there are forward–looking variables in the model **and** the rank condition is satisfied.

First order approximation, XI

Reduced form solution

- Finally, we have:

$$\hat{y}_t = g_y \hat{y}_{t-1} + g_u \epsilon_t$$

$$\Leftrightarrow y_t = (I_n - g_y) y^\star + g_y y_{t-1} + g_u \epsilon_t$$

- The unconditional expectation of y_t is the deterministic steady state, $\mathbb{E}\left[y_t\right] = y^\star$. This is a manifestation of the certainty equivalence property

- The unconditional covariance matrix, $\Sigma_y = \mathbb{V}\left[y_t\right]$, must solve:

$$\Sigma_y = g_y \Sigma_y g_y' + g_u \Sigma_\epsilon g_u'$$

Specialized algorithms exist to solve efficiently this kind of equations... Otherwise the vec operator and kronecker product can be used (not efficient):

$$\mathrm{vec}\Sigma_y = \left(I_{n^2} - g_y \otimes g_y\right)^{-1} \mathrm{vec} g_u \Sigma_\epsilon g_u'$$

First order approximation, XII

Reduced form solution

- Inverting the reduced form, we obtain the MA(∞) representation:

$$\Leftrightarrow y_t = y^\star + \sum_{i=0}^{\infty} g_y^i g_u \epsilon_{t-i}$$

- Let e_j be the j-th column of I_n
- The sequence $\{g_y^i g_u e_j\}_{i=0}^{\infty}$ is the IRF associated to a unitary shock on the j-th innovation
- If the innovations are not orthogonal (which is a bad practice) a Cholesky decomposition can be used.

Let's play!

```
model;

    1/C = beta*(1/C(+1))*(alpha*A(1)*K^(alpha-1)*N(1)^(1-alpha) + (1-delta));

    B*N^(gamma) = (1/C)*(1-alpha)*A*K(-1)^(alpha)*N^(-alpha);

    Y = A*K(-1)^(alpha)*N^(1-alpha);

    K = I + (1-delta)*K(-1);

    Y = C + I + G;

    G = GY*Y;

    log(A) = rhoA*log(A(-1)) + eA;

    log(GY) = (1-rhoGY)*log(GYstar) + rhoGY*log(GY(-1)) + eG;

    log(B) = (1-rhoB)*log(Bstar) + rhoB*log(B(-1)) + eB;

end;
```

Let's play!

- Do you recognize the model?
- Write a complete `mod` file for this model
- Compute the steady state (with a nonlinear solver and with a closed form solution)
- Simulate the model