

# Model-consistent-expectations in the ECB-BASE: First results and Roadmap

Stéphane Adjemian   Nikola Bokan   Matthieu Darracq Pariès

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# Context

## MCE activities within the ECB-MC project

- Starts from the ECB-BASE (backward) model and introduces further developments
  - new "infrastructure": one-stop shop allowing
    - pre-estimation data-generation
    - estimation
    - post-estimation data generation
    - dynare .mod model production
    - model simulations
  - re-estimation of the model on recent euro area data (ECB-BASE V2)
  - appropriate convergence towards the balance growth path
  - various other (bug) fixes

# ECB-BASE V2 (beta)

Full re-estimation using the recent EA data

- First draft of ECB-BASE V2 pointed out to potential issues in several blocks
- Improved properties by fine-tuning towards ECB-BASE V1 along several dimensions [▶ V1 vs V2 IRFs](#)
  - Consumption block
    - BOLS estimation of the target equation
    - interest rate coeff in the PAC equation constrained as in ECB-BASE V1 (WP)
  - BOLS estimation of wage target equation
  - BOLS estimation of trade equations
- Some further work is still needed to fine tune the ECB-BASE V2 and make it operational in the (B)MPE process
- MCE activities will proceed in parallel to this process

# Infrastructures: V1 vs V2

Figure 2: Short-term interest rate shock (100bp)



# Infrastructures: V1 vs V2

Figure 3: Term premium shock (100bp)



# Outline

- 1 Dealing with expectations
  - VAR expectations
  - PAC equation
- 2 MCE specification
  - Exchange rate determination
  - Financial block
  - Price and wage setting blocks
- 3 Simulations
  - Backward versus full MCE
  - Standard shocks across MCE specifications
  - Forward guidance shocks
  - Anticipated productivity shocks
- 4 Roadmap for MCE activities

# Expectations in ECB BASE

## VAR expectations

- ECB BASE is estimated under the assumption that agents use a VAR model to form expectations.
- Consumption depends on permanent income(s):

$$y_t^P = \sum_{h=0}^{\infty} \beta^h y_{t+h}$$

the flow of expected incomes can be computed from a VAR.  
Suppose  $y$  is an element of  $Y$  and that:

$$Y_t = AY_{t-1} + \epsilon_t$$

Then we have:

$$y_{t+h} = ZA^h Y_t \quad \text{and} \quad y_t^P = Z \left( \sum_{h=0}^{\infty} \beta^h A^h \right) Y_t = Z (I - \beta A)^{-1} Y_t$$

# Expectations in ECB BASE

## VAR expectations (contd')

- Implementation in Dynare is straightforward:
  - Declare a VAR model with the `VAR_MODEL` command (pointing to some equations in the `model` block),
  - Alternatively, declare a trend component model with the `TREND_COMPONENT_MODEL` command,
  - Use the `VAR_EXPECTATION_MODEL` command to specify the expected term to be used in the `model` block (which variable or linear combination of variables is to be expected, at which horizon, ...),
  - Use the `VAR_EXPECTATION` command in the `model` block.



# Expectations in ECB BASE

## VAR expectations in WAPRO

```

trend_component_model(model_name = base_var_model,
eqtags = ['U2_WAPRO_A_YED', 'U2_WAPRO_G_YER', 'U2_WAPRO_ESTN',
          'U2_WAPRO_EHIC', 'U2_WAPRO_G_EYER', 'U2_WAPRO_STN'],
targets=['U2_WAPRO_G_EYER', 'U2_WAPRO_EHIC', 'U2_WAPRO_ESTN']);

var_expectation_model(model_name = U2_A_YED_OP_FCAST,
expression = U2_WAPRO_A_YED,
auxiliary_model_name = base_var_model,
horizon = 1,
discount = 1.0);

model;
...
U2_A_YED = (0.387271924091236*U2_A_YED(-1)
+ 0.629532709573858*var_expectation(U2_A_YED_OP_FCAST)
+ (1-0.629532709573858)*(1-0.387271924091236)*U2_EHIC
+ 0.421357609729608*(U2_G_C_CER + 0.3/(1-0.3)*U2_G_YER))
/ (1+0.387271924091236*0.629532709573858) + res_U2_A_YED;

end;
```

# PAC equation

- Penalize deviations of a variable from its target and the variations of the variable:

$$C_t = \sum_{i=0}^{\infty} \beta^i \left[ (y_{t+i} - y_{t+i}^*)^2 + \sum_{k=1}^m b_k (\Delta^k y_{t+i})^2 \right]$$

- Reduced form associated to the minimization of the cost  $C_t$ :

$$\Delta y_t = a_0(y_{t-1}^* - y_{t-1}) + \sum_{k=1}^{m-1} a_k \Delta y_{t-k} + \underbrace{\sum_{j=0}^{\infty} d_j \Delta y_{t+j}^*}_{\mathcal{Z}_t}$$

Expected variations  
of the target

- Where  $\{a_i\}_{i=0}^{m-1}$  and  $\{d_i\}_{i \in \mathbb{N}}$  are reduced form parameters depending on the parameters of the cost function.

# PAC equation

## Estimation

- The estimation of the parameters  $\{a_i\}_{i=0}^{m-1}$  (not the deep parameters) requires the knowledge of  $\mathcal{Z}_t$  (expectations).
- The estimation is done under the assumption that the expectations are VAR based.
- Each  $\Delta y_{t+i}$  (for  $i = , \dots, \infty$ ) can be expressed as a linear combination of the VAR variables, where the weights are a convolution of the parameters  $\{a_i\}_{i=0}^{m-1}$  and the VAR autoregressive matrices.

$$\Delta y_t = a_0(y_{t-1}^* - y_{t-1}) + \sum_{k=1}^{m-1} a_k \Delta y_{t-k} + h' Y_t + \epsilon_t$$

- Estimation is carried through NLS.

# PAC equation

## MCE simulations

- In the MCE version of ECB-BASE we cannot rely on a VAR model to determine the expected variations of the target,  $\mathcal{Z}_t$ .
- Rewrite the expected infinite sum  $\mathcal{Z}_t$  recursively:

$$\mathcal{Z}_t = - \sum_{i=1}^m \alpha_i \beta^i \mathcal{Z}_{t+i} + \left( 1 + \sum_{i=1}^m \alpha_i \right) \times \left[ \Delta y_t^* + \sum_{k=1}^{m-1} \left( \sum_{j=k}^{m-1} \alpha_{j+1} \beta^{j+1} \Delta y_{t+k}^* \right) \right]$$

- The parameters  $\{\alpha_i\}_{i=1}^m$  depend on the previously estimated parameters  $\{\alpha_i\}_{i=0}^{m-1}$ .

# PAC equation

## PAC with Dynare

- Dynare expands  $\mathcal{Z}_t$  (depending on the context) for you and estimate the PAC equation.

```
var_model(model_name=toto, eqtags=['eq:x', 'eq:y']);
pac_model(auxiliary_model_name=toto,
discount=beta, model_name=pacman);
model;
[name='eq:y']
y = a_y_1*y(-1) + a_y_2*diff(x(-1)) + b_y_1*y(-2)
+ b_y_2*diff(x(-2)) + ey ;
[name='eq:x']
diff(x) = b_x_1*y(-2) + b_x_2*diff(x(-1)) + ex ;

[name='eq:pac']
diff(z) = e_c_m*(x(-1)-z(-1)) + c_z_1*diff(z(-1))
+ c_z_2*diff(z(-2))
+ pac_expectation(pacman) + ez;
end;
```

- For the MCE just drop option `auxiliary_model_name`

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# MCE specification: Exchange rate determination

## MCE for the Uncovered Interest rate Parity equation (UIP)

- Replacing the "level" equation for the nominal exchange rate in the backward model by a UIP condition
- The expected depreciation rate depends on the short-term interest rate differential...
- ... augmented with a term premium effect to obtain a plausible "APP-like" transmission of term premium shocks
- Given the consumption specification, adding a term on NFA in the UIP is not necessary to ensure stationary NFA dynamics

*Empirical and theoretical challenges remain on the UIP specification*

# MCE specification: financial block

## MCE in the term structure of interest rate

- Long-term interest rates
  - Expectation theory for the risk-free 10-year OIS rate: we introduce a consol bond serving geometrically decaying coupons, discounted by the short-term policy rate and a duration corresponding the one of a 10-year zero coupon.
  - The 10-year OIS rate accounting for a term premium: we introduce a similar consol with a discounting of coupons augmented by a term premium.
  - Similar treatment for corporate bond



# MCE specification: financial block

## MCE in financial spreads

- Replacing the VAR-based expectation by MCE expectation for 10-year average of expected output gap
- this variable then loads into the financial spreads (term premium, corporate spread, lending rate spreads and cost-of-equity)
- Consistent reformulation of the revaluation effects on households net financial wealth

*Follow-up: stock prices may be formally specified*

# MCE specification: wapro and wage blocks

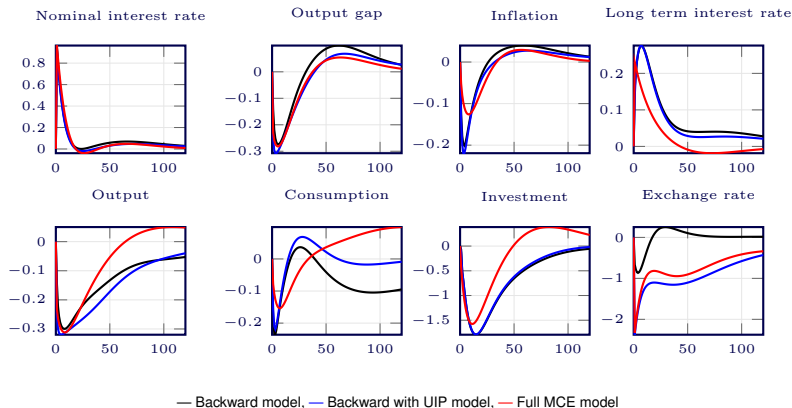
## MCE in the wage and price setting equations

- Replacing the VAR-based expectation by MCE expectation for the next period inflation term in the GDP-deflator equation
- Replacing the VAR-based expectation by MCE expectation for the next period wage gap term in the wage gap equation
- Long-term inflation expectations remain imperfectly anchored as in the backward model

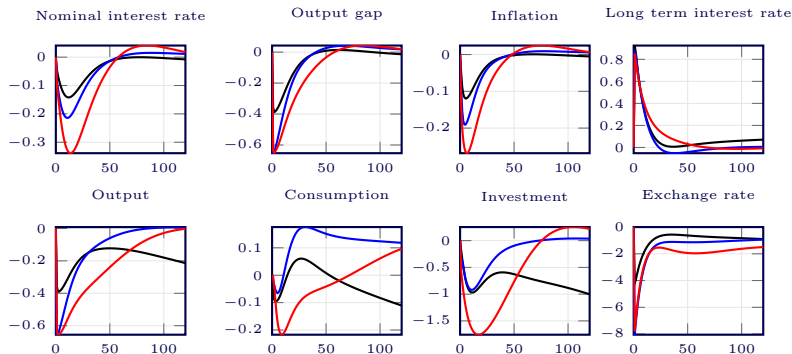
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# Responses to a one point shock on the nominal interest rate

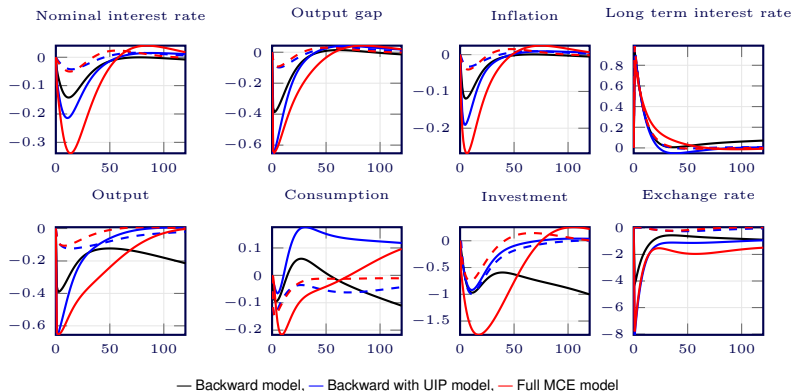


# Responses to a one point shock on the term premium, I



— Backward model, — Backward with UIP model, — Full MCE model

# Responses to a one point shock on the term premium, II



dashed lines are simulated from the same models without TP in exchange rate equation

# Simulations

- Backward model with UIP,
- Full MCE,
- Hybrid mode (MCE in wage, wapro, financial and exchange rate blocks).

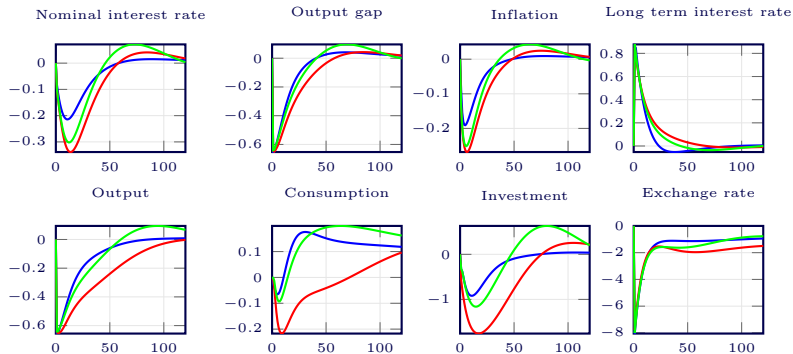
# Responses to a one point shock on the nominal interest rate



— Backward with UIP model, — Full MCE model, — Hybrid model

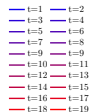
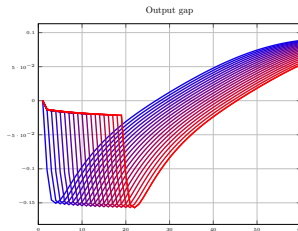


# Responses to a one point shock on the term premium

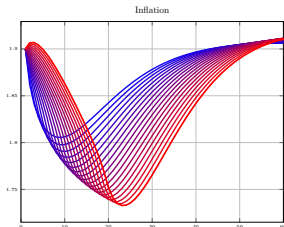
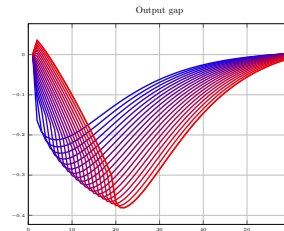
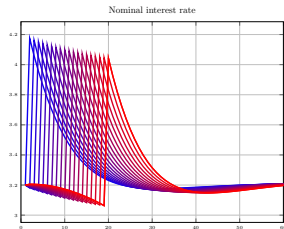


— Backward with UIP model, — Full MCE model, — Hybrid model

# Forward guidance (Backward with UIP model)

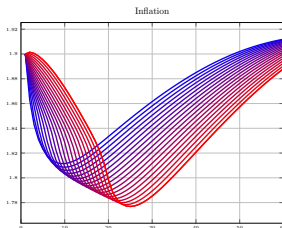
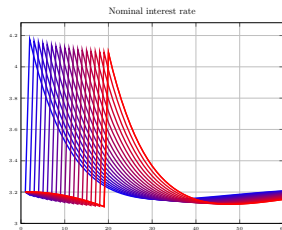


# Forward guidance (full MCE)



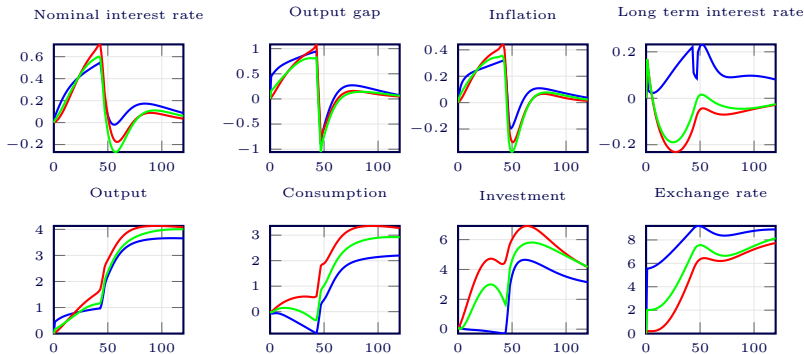
t=1 t=2  
t=3 t=4  
t=5 t=6  
t=7 t=8  
t=9 t=9  
t=10 t=11  
t=12 t=13  
t=14 t=15  
t=16 t=17  
t=18 t=19

# Forward guidance (Hybrid model)



— t=1 — t=2  
— t=3 — t=4  
— t=5 — t=6  
— t=7 — t=8  
— t=9 — t=9  
— t=10 — t=11  
— t=12 — t=13  
— t=14 — t=15  
— t=16 — t=17  
— t=18 — t=19

# Expected TFP growth shock in 10 years for one year ( $4 \times 1\%$ )



— Backward with UIP model, — Full MCE model, — Hybrid model

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# Follow-up activities

## Possible workstreams

- Codes and Documentation of the "as-is" MCE version of ECB-BASE
- Stationary version of the model and linear approximation
  - Tractability of stochastic simulations
  - Stabilisation properties of alternative monetary policy conduct
- Other policy analysis
  - Fiscal-Monetary interactions
- Empirical validation of the MCE model
  - Filtering
  - Indirect inference on the Wapro/wage blocks
  - System-wide inference using the linear model