Model-consistent-expectations in the ECB-BASE: First results and Roadmap

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October, 2020

Context

MCE activities within the ECB-MC project

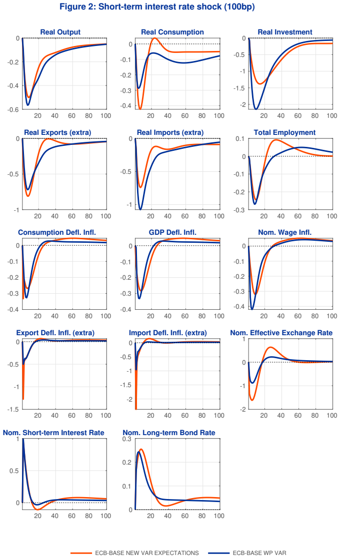
- Starts from the ECB-BASE (backward) model and introduces further developments
  - new "infrastructure": one-stop shop allowing
    - pre-estimation data-generation
    - estimation
    - post-estimation data generation
    - dynare .mod model production
    - model simulations
  - re-estimation of the model on recent euro area data (ECB-BASE V2)
  - appropriate convergence towards the balance growth path
  - various other (bug) fixes

ECB-BASE V2 (beta)

Full re-estimation using the recent EA data

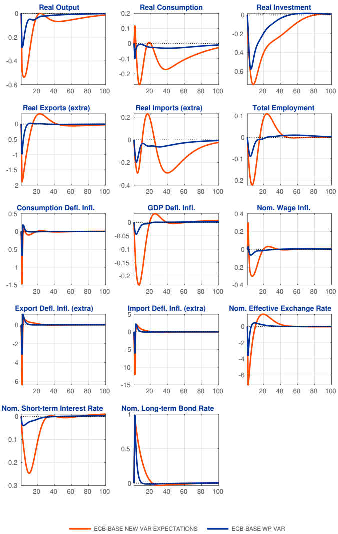
- First draft of ECB-BASE V2 pointed out to potential issues in several blocks
- Improved properties by fine-tuning towards ECB-BASE V1 along several dimensions ▸ V1 vs V2 IRFs
  - Consumption block
    - BOLS estimation of the target equation
    - interest rate coeff in the PAC equation constrained as in ECB-BASE V1 (WP)
  - BOLS estimation of wage target equation
  - BOLS estimation of trade equations
- Some further work is still needed to fine tune the ECB-BASE V2 and make it operational in the (B)MPE process
- MCE activities will proceed in parallel to this process

Infrastuctures: V1 vs V2



Infrastuctures: V1 vs V2

Figure 3: Term premium shock (100bp)



Outline

Expectations in ECB BASE

VAR expectations

- ECB BASE is estimated under the assumption that agents use a VAR model to form expectations.

- Consumption depends on permanent income(s):

$$y_t^P = \sum_{h=0}^{\infty} \beta^h y_{t+h}$$

the flow of expected incomes can be computed from a VAR. Suppose y is an element of Y and that:

$$Y_t = AY_{t-1} + \epsilon_t$$

Then we have:

$$y_{t+h} = ZA^h Y_t \quad \text{and} \quad y_t^P = Z\left(\sum_{h=0}^{\infty} \beta^h A^h\right) Y_t = Z\left(I - \beta A\right)^{-1} Y_t$$

Expectations in ECB BASE

VAR expectations (contd')

- Implementation in Dynare is straightforward:
  - Declare a VAR model with the `VAR_MODEL` command (pointing to some equations in the `model` block),
  - Alternatively, declare a trend component model with the `TREND_COMPONENT_MODEL` command,
  - Use the `VAR_EXPECTATION_MODEL` command to specify the expected term to be used in the `model` block (which variable or linear combination of variables is to be expected, at which horizon, . . .),
  - Use the `VAR_EXPECTATION` command in the `model` block.

Expectations in ECB BASE

VAR expectations in WAPRO

```
trend_component_model(model_name = base_var_model,
eqtags = ['U2_WAPRO_A_YED', 'U2_WAPRO_G_YER', 'U2_WAPRO_ESTN',
          'U2_WAPRO_EHIC', 'U2_WAPRO_G_EYER', 'U2_WAPRO_STN'],
targets=['U2_WAPRO_G_EYER', 'U2_WAPRO_EHIC', 'U2_WAPRO_ESTN']);

var_expectation_model(model_name = U2_A_YED_OP_FCAST,
expression = U2_WAPRO_A_YED,
auxiliary_model_name = base_var_model,
horizon = 1,
discount = 1.0);

model;
...
U2_A_YED = (0.387271924091236*U2_A_YED(-1)
+ 0.629532709573858*var_expectation(U2_A_YED_OP_FCAST)
+ (1-0.629532709573858)*(1-0.387271924091236)*U2_EHIC
+ 0.421357609729608*(U2_G_C_CER + 0.3/(1-0.3)*U2_G_YER))
/ (1+0.387271924091236*0.629532709573858)+ res_U2_A_YED;

end;
```

PAC equation

- Penalize deviations of a variable from its target and the variations of the variable:

$$C_t = \sum_{i=0}^{\infty} \beta^i \left[(y_{t+i} - y^\star_{t+i})^2 + \sum_{k=1}^{m} b_k \left(\Delta^k y_{t+i} \right)^2 \right]$$

- Reduced form associated to the minimization of the cost C_t:

$$\Delta y_t = a_0(y^\star_{t-1} - y_{t-1}) + \sum_{k=1}^{m-1} a_k \Delta y_{t-k} + \underbrace{\sum_{j=0}^{\infty} d_j \Delta y^\star_{t+j}}_{\substack{\mathcal{Z}_t \\ \text{Expected variations} \\ \text{of the target}}}$$

- Where $\{a_i\}_{i=0}^{m-1}$ and $\{d_i\}_{i\in\mathbb{N}}$ are reduced form parameters depending on the parameters of the cost function.

PAC equation

Estimation

- The estimation of the parameters $\{a_i\}_{i=0}^{m-1}$ (not the deep parameters) requires the knowledge of $\mathcal{Z}_t$ (expectations).
- The estimation is done under the assumption that the expectations are VAR based.
- Each Δy_{t+i} (for $i =, \dots, \infty$) can be expressed as a linear combination of the VAR variables, where the weights are a convolution of the parameters $\{a_i\}_{i=0}^{m-1}$ and the VAR autorgressive matrices.

$$\Delta y_t = a_0(y^\star_{t-1} - y_{t-1}) + \sum_{k=1}^{m-1} a_k \Delta y_{t-k} + h' Y_t + \epsilon_t$$

- Estimation is carried through NLS.

PAC equation
MCE simulations

- In the MCE version of ECB-BASE we cannot rely on a VAR model to determine the expected variations of the target, $\mathcal{Z}_t$.

- Rewrite the expected infinite sum $\mathcal{Z}_t$ recursively:

$$\mathcal{Z}_t = -\sum_{i=1}^{m} \alpha_i \beta^i \mathcal{Z}_{t+i} + \left(1 + \sum_{i=1}^{m} \alpha_i\right) \times \left[\Delta y_t^\star + \sum_{k=1}^{m-1} \left(\sum_{j=k}^{m-1} \alpha_{j+1} \beta^{j+1} \Delta y_{t+k}^\star\right)\right]$$

- The parameters $\{\alpha_i\}_{i=1}^{m}$ depend on the previously estimated parameters $\{a_i\}_{i=0}^{m-1}$.

PAC equation

PAC with Dynare

- Dynare expands $\mathcal{Z}_t$ (depending on the context) for you and estimate the PAC equation.

```
var_model(model_name=toto, eqtags=['eq:x', 'eq:y']);
pac_model(auxiliary_model_name=toto,
discount=beta, model_name=pacman);
model;
[name='eq:y']
y = a_y_1*y(-1) + a_y_2*diff(x(-1)) + b_y_1*y(-2)
+ b_y_2*diff(x(-2)) + ey ;
[name='eq:x']
diff(x) = b_x_1*y(-2) + b_x_2*diff(x(-1)) + ex ;

[name='eq:pac']
diff(z) = e_c_m*(x(-1)-z(-1)) + c_z_1*diff(z(-1))
+ c_z_2*diff(z(-2))
+ pac_expectation(pacman) + ez;
end;
```

- For the MCE just drop option `auxiliary_model_name`

Outline

MCE specification: Exchange rate determination

MCE for the Uncovered Interest rate Parity equation (UIP)

- Replacing the ”level” equation for the nominal exchange rate in the backward model by a UIP condition
- The expected depreciation rate depends on the short-term interest rate differential...
- ... augmented with a term premium effect to obtain a plausible ”APP-like” transmission of term premium shocks
- Given the consumption specification, adding a term on NFA in the UIP is not necessary to ensure stationary NFA dynamics

Empirical and theoretical challenges remain on the UIP specification

MCE specification: financial block

MCE in the term structure of interest rate

- Long-term interest rates
  - Expectation theory for the risk-free 10-year OIS rate: we introduce a consol bond serving geometrically decaying coupons, discounted by the short-term policy rate and a duration corresponding the one of a 10-year zero coupon.
  - The 10-year OIS rate accounting for a term premium: we introduce a similar consol with a discounting of coupons augmented by a term premium.
  - Similar treatment for corporate bond

MCE specification: financial block

MCE in financial spreads

- Replacing the VAR-based expectation by MCE expectation for 10-year average of expected output gap
- this variable then loads into the financial spreads (term premium, corporate spread, lending rate spreads and cost-of-equity)
- Consistent reformulation of the revaluation effects on households net financial wealth

Follow-up: stock prices may be formally specified

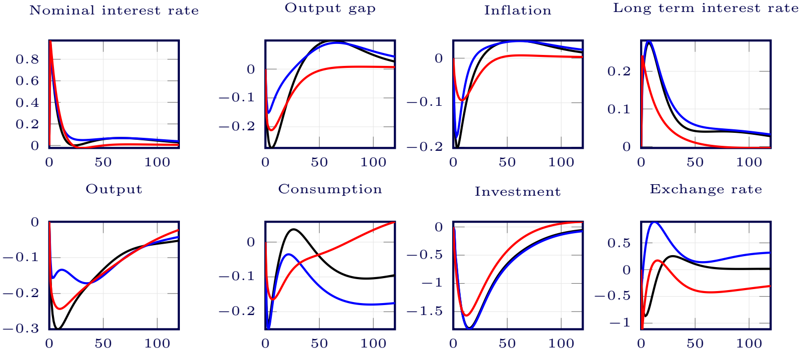
MCE specification: wapro and wage blocks

MCE in the wage and price setting equations

- Replacing the VAR-based expectation by MCE expectation for the next period inflation term in the GDP-deflator equation
- Replacing the VAR-based expectation by MCE expectation for the next period wage gap term in the wage gap equation
- Long-term inflation expectations remain imperfectly anchored as in the backward model

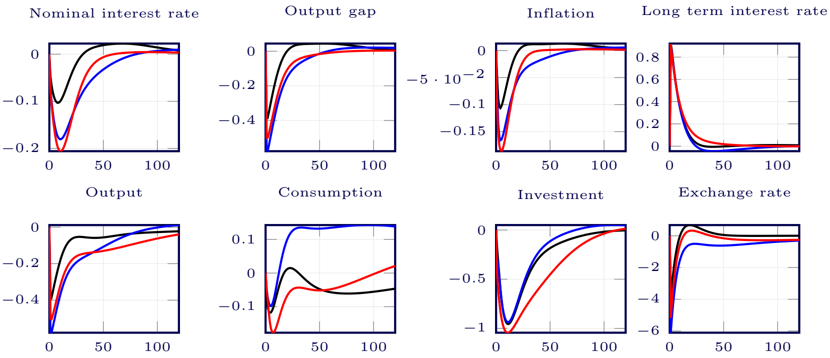
Outline

Responses to a one point shock on the nominal interest rate



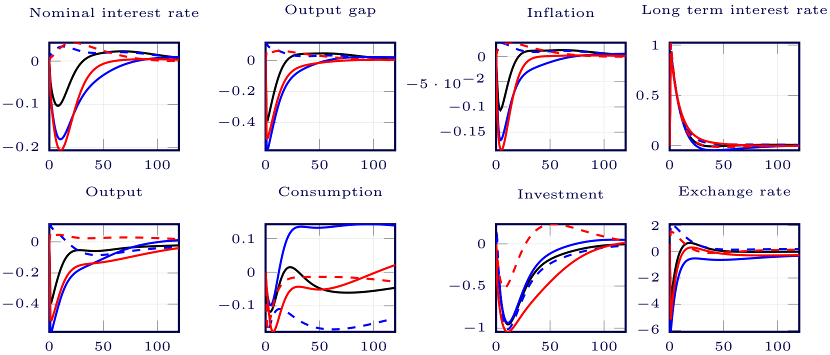
— Backward model, — Backward with UIP model, — Full MCE model

Responses to a one point shock on the term premium, I



— Backward model, — Backward with UIP model, — Full MCE model

Responses to a one point shock on the term premium, II



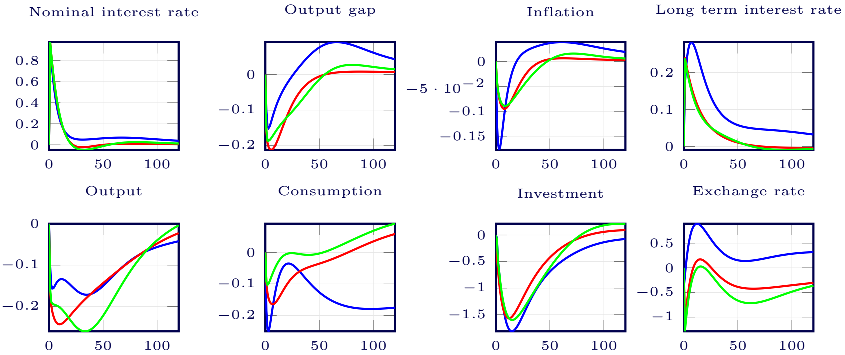
— Backward model, — Backward with UIP model, — Full MCE model

dashed lines are simulated from the same models without TP in exchange rate equation

Simulations

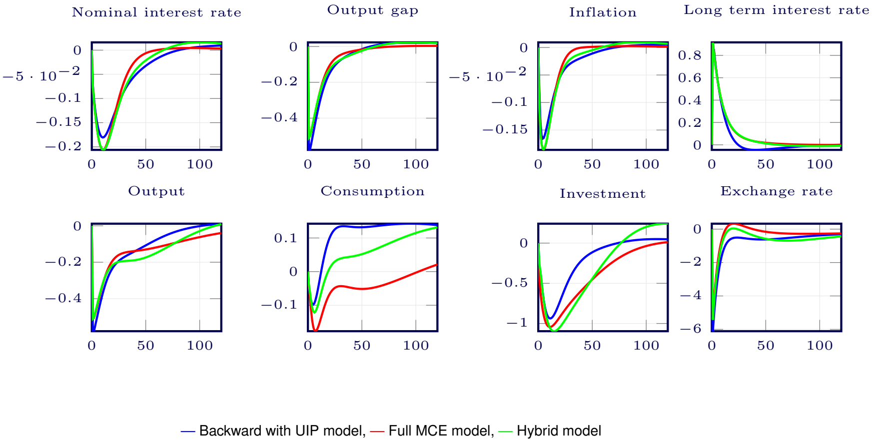
- — Backward model with UIP,
- — Full MCE,
- — Hybrid mode (MCE in wage, wapro, financial and exchange rate blocks).

Responses to a one point shock on the nominal interest rate

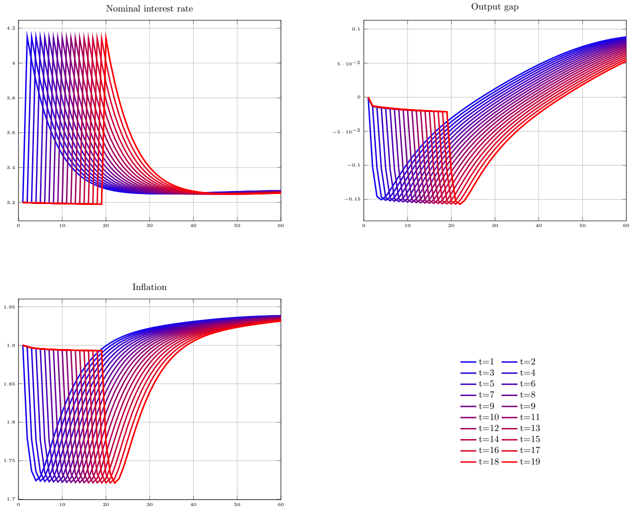


— Backward with UIP model, — Full MCE model, — Hybrid model

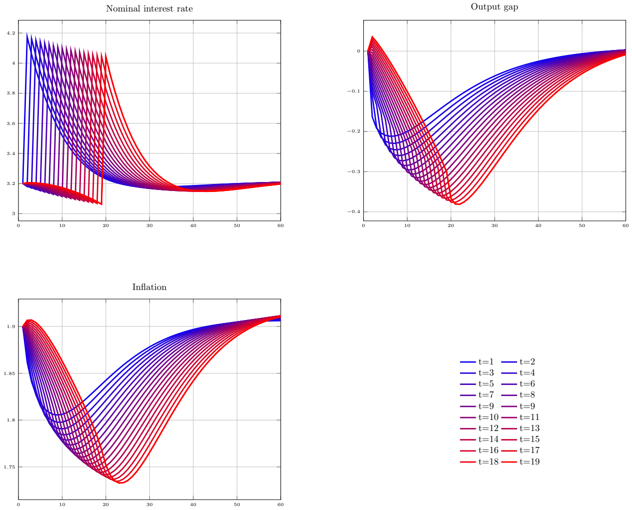
Responses to a one point shock on the term premium



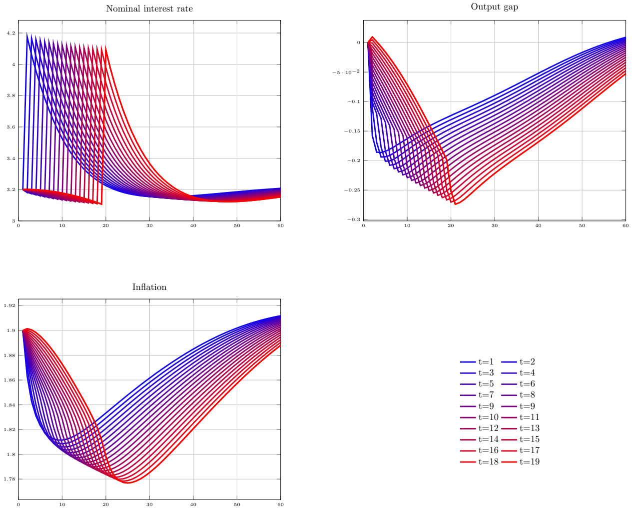
Forward guidance (Backward with UIP model)



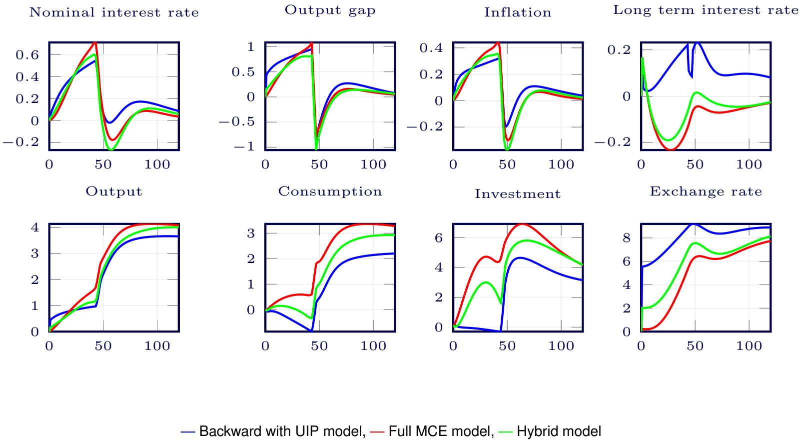
Forward guidance (full MCE)



Forward guidance (Hybrid model)



Expected TFP growth shock in 10 years for one year (4×1%)



Outline

1. Dealing with expectations
   - VAR expectations
   - PAC equation
2. MCE specification
   - Exchange rate determination
   - Financial block
   - Price and wage setting blocks
3. Simulations
   - Backward versus full MCE
   - Standard shocks across MCE specifications
   - Forward guidance shocks
   - Anticipated productivity shocks
4. Roadmap for MCE activities

Follow-up activities

Possible workstreams

- Codes and Documentation of the ”as-is” MCE version of ECB-BASE
- Stationary version of the model and linear approximation
  - Tractability of stochastic simulations
  - Stabilisation properties of alternative monetary policy conduct
- Other policy analysis
  - Fiscal-Monetary interactions
- Empirical validation of the MCE model
  - Filtering
  - Indirect inference on the Wapro/wage blocks
  - System-wide inference using the linear model