

Model-consistent-expectations in the ECB-BASE: First results and Roadmap

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Context

MCE activities within the ECB-MC project

- Starts from the ECB-BASE (backward) model and introduces further developments
 - new "infrastructure": one-stop shop allowing
 - pre-estimation data-generation
 - estimation
 - post-estimation data generation
 - dynare .mod model production
 - model simulations
 - re-estimation of the model on recent euro area data (ECB-BASE V2)
 - appropriate convergence towards the balance growth path
 - various other (bug) fixes

ECB-BASE V2 (beta)

Full re-estimation using the recent EA data

- First draft of ECB-BASE V2 pointed out to potential issues in several blocks
- Improved properties by fine-tuning towards ECB-BASE V1 along several dimensions
 - Consumption block
 - BOLS estimation of the target equation
 - interest rate coeff in the PAC equation constrained as in ECB-BASE V1 (WP)
 - BOLS estimation of wage target equation
 - BOLS estimation of trade equations
- Some further work is still needed to fine tune the ECB-BASE V2 and make it operational in the (B)MPE process
- MCE activities will proceed in parallel to this process

Outline

- 1 Dealing with expectations
 - VAR expectations
 - PAC equation
- 2 MCE specification
 - Exchange rate determination
 - Financial block
 - Price and wage setting blocks
- 3 Simulations
 - Benchmark backward model
 - Backward versus full MCE
 - Standard shocks across MCE specifications
 - Forward guidance shocks
 - Anticipated productivity shocks
- 4 Roadmap for MCE activities

Expectations in ECB BASE

VAR expectations

- ECB BASE is estimated under the assumption that agents use a VAR model to form expectations.
- Consumption depends on permanent income(s):

$$y_t^P = \sum_{h=0}^{\infty} \beta^h y_{t+h}$$

the flow of expected incomes can be computed from a VAR.
Suppose y is an element of Y and that:

$$Y_t = AY_{t-1} + \epsilon_t$$

Then we have:

$$y_{t+h} = ZA^h Y_t \quad \text{and} \quad y_t^P = Z \left(\sum_{h=0}^{\infty} \beta^h A^h \right) Y_t = Z (I - \beta A)^{-1} Y_t$$

Expectations in ECB BASE

VAR expectations (contd')

- Implementation in Dynare is straightforward:
 - Declare a VAR model with the `VAR_MODEL` command (pointing to some equations in the `model` block),
 - Alternatively, declare a trend component model with the `TREND_COMPONENT_MODEL` command,
 - Use the `VAR_EXPECTATION_MODEL` command to specify the expected term to be used in the `model` block (which variable or linear combination of variables is to be expected, at which horizon, ...),
 - Use the `VAR_EXPECTATION` command in the `model` block.

Expectations in ECB BASE

VAR expectations in WAPRO

```

trend_component_model(model_name = base_var_model,
eqtags = ['U2_WAPRO_A_YED', 'U2_WAPRO_G_YER', 'U2_WAPRO_ESTN',
          'U2_WAPRO_EHIC', 'U2_WAPRO_G_EYER', 'U2_WAPRO_STN'],
targets=['U2_WAPRO_G_EYER', 'U2_WAPRO_EHIC', 'U2_WAPRO_ESTN']);

var_expectation_model(model_name = U2_A_YED_OP_FCAST,
expression = U2_WAPRO_A_YED,
auxiliary_model_name = base_var_model,
horizon = 1,
discount = 1.0);

model;
...
U2_A_YED = (0.387271924091236*U2_A_YED(-1)
+ 0.629532709573858*var_expectation(U2_A_YED_OP_FCAST)
+ (1-0.629532709573858)*(1-0.387271924091236)*U2_EHIC
+ 0.421357609729608*(U2_G_C_CER + 0.3/(1-0.3)*U2_G_YER))
/ (1+0.387271924091236*0.629532709573858)+ res_U2_A_YED;

end;
```

PAC equation

- Penalize deviations of a variable from its target and the variations of the variable:

$$C_t = \sum_{i=0}^{\infty} \beta^i \left[(y_{t+i} - y_{t+i}^*)^2 + \sum_{k=1}^m b_k (\Delta^k y_{t+i})^2 \right]$$

- Reduced form associated to the minimization of the cost C_t :

$$\Delta y_t = a_0(y_{t-1}^* - y_{t-1}) + \sum_{k=1}^{m-1} a_k \Delta y_{t-k} + \underbrace{\sum_{j=0}^{\infty} d_j \Delta y_{t+j}^*}_{\mathcal{Z}_t}$$

Expected variations
of the target

- Where $\{a_i\}_{i=0}^{m-1}$ and $\{d_i\}_{i \in \mathbb{N}}$ are reduced form parameters depending on the parameters of the cost function.

PAC equation

Estimation

- The estimation of the parameters $\{a_i\}_{i=0}^{m-1}$ (not the deep parameters) requires the knowledge of \mathcal{Z}_t (expectations).
- The estimation is done under the assumption that the expectations are VAR based.
- Each Δy_{t+i} (for $i = , \dots, \infty$) can be expressed as a linear combination of the VAR variables, where the weights are a convolution of the parameters $\{a_i\}_{i=0}^{m-1}$ and the VAR autoregressive matrices.

$$\Delta y_t = a_0(y_{t-1}^* - y_{t-1}) + \sum_{k=1}^{m-1} a_k \Delta y_{t-k} + h' Y_t + \epsilon_t$$

- Estimation is carried through NLS.

PAC equation

MCE simulations

- In the MCE version of ECB-BASE we cannot rely on a VAR model to determine the expected variations of the target, \mathcal{Z}_t .
- Rewrite the expected infinite sum \mathcal{Z}_t recursively:

$$\mathcal{Z}_t = - \sum_{i=1}^m \alpha_i \beta^i \mathcal{Z}_{t+i} + \left(1 + \sum_{i=1}^m \alpha_i \right) \times \left[\Delta y_t^* + \sum_{k=1}^{m-1} \left(\sum_{j=k}^{m-1} \alpha_{j+1} \beta^{j+1} \Delta y_{t+k}^* \right) \right]$$

- The parameters $\{\alpha_i\}_{i=1}^m$ depend on the previously estimated parameters $\{\alpha_i\}_{i=0}^{m-1}$.

PAC equation

PAC with Dynare

- Dynare expands \mathcal{Z}_t (depending on the context) for you and estimate the PAC equation.

```
var_model(model_name=toto, eqtags=['eq:x', 'eq:y']);
pac_model(auxiliary_model_name=toto,
discount=beta, model_name=pacman);
model;
[name='eq:y']
y = a_y_1*y(-1) + a_y_2*diff(x(-1)) + b_y_1*y(-2)
+ b_y_2*diff(x(-2)) + ey ;
[name='eq:x']
diff(x) = b_x_1*y(-2) + b_x_2*diff(x(-1)) + ex ;

[name='eq:pac']
diff(z) = e_c_m*(x(-1)-z(-1)) + c_z_1*diff(z(-1))
+ c_z_2*diff(z(-2))
+ pac_expectation(pacman) + ez;
end;
```

- For the MCE just drop option `auxiliary_model_name`

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MCE specification: Exchange rate determination

MCE for the Uncovered Interest rate Parity equation (UIP)

- Replacing the "level" equation for the nominal exchange rate in the backward model by a UIP condition
- The expected depreciation rate depends on the short-term interest rate differential...
- ... augmented with a term premium effect to obtain a plausible "APP-like" transmission of term premium shocks
- Given the consumption specification, adding a term on NFA in the UIP is not necessary to ensure stationary NFA dynamics

Empirical and theoretical challenges remain on the UIP specification

MCE specification: financial block

MCE in the term structure of interest rate

- Long-term interest rates
 - Expectation theory for the risk-free 10-year OIS rate: we introduce a consol bond serving geometrically decaying coupons, discounted by the short-term policy rate and a duration corresponding the one of a 10-year zero coupon.
 - The 10-year OIS rate accounting for a term premium: we introduce a similar consol with a discounting of coupons augmented by a term premium.
 - Similar treatment for corporate bond

MCE specification: financial block

MCE in financial spreads

- Replacing the VAR-based expectation by MCE expectation for 10-year average of expected output gap
- this variable then loads into the financial spreads (term premium, corporate spread, lending rate spreads and cost-of-equity)
- Consistent reformulation of the revaluation effects on households net financial wealth

Follow-up: stock prices may be formally specified

MCE specification: wapro and wage blocks

MCE in the wage and price setting equations

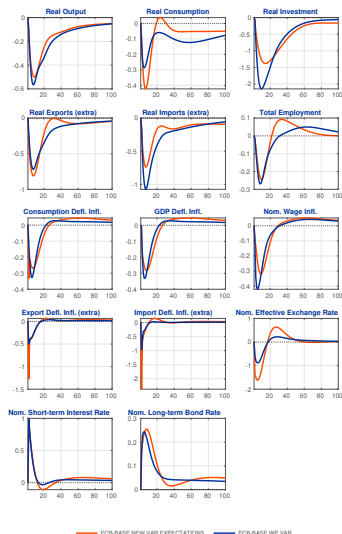
- Replacing the VAR-based expectation by MCE expectation for the next period inflation term in the GDP-deflator equation
- Replacing the VAR-based expectation by MCE expectation for the next period wage gap term in the wage gap equation
- Long-term inflation expectations remain imperfectly anchored as in the backward model

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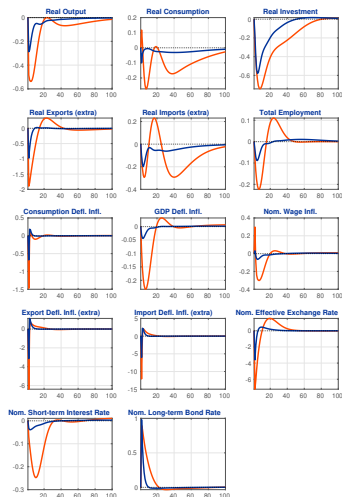
Infrastructures: V1 vs V2

Figure 2: Short-term interest rate shock (100bp)

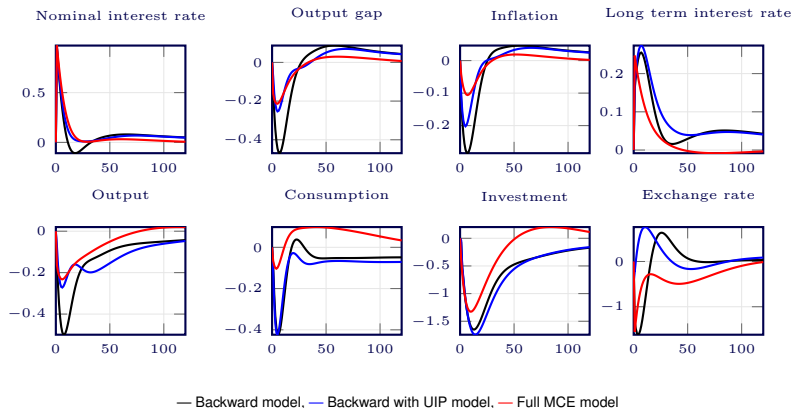


Infrastructures: V1 vs V2

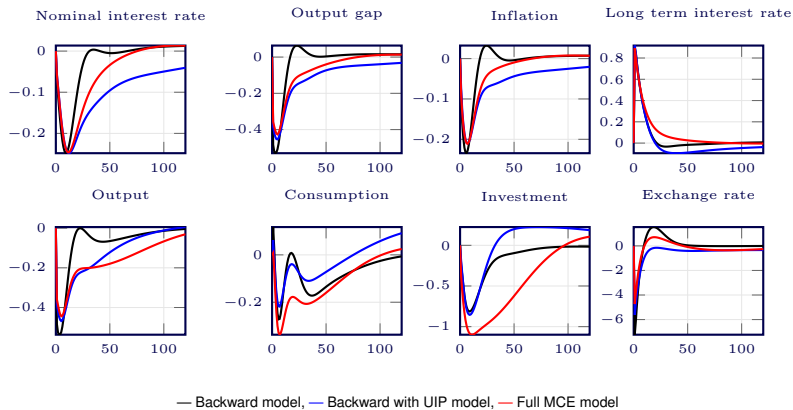
Figure 3: Term premium shock (100bp)



Responses to a one point shock on the nominal interest rate



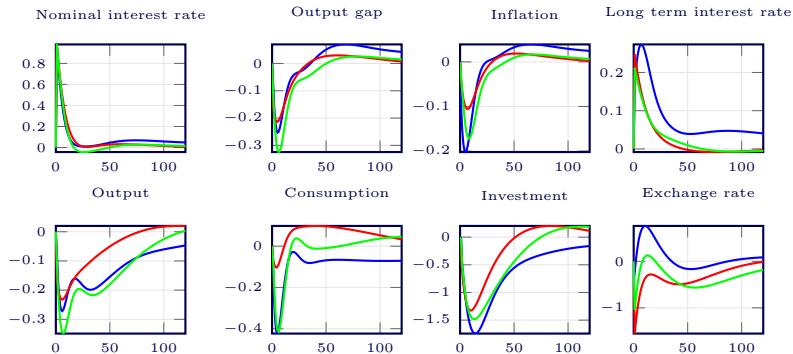
Responses to a one point shock on the term premium



Simulations

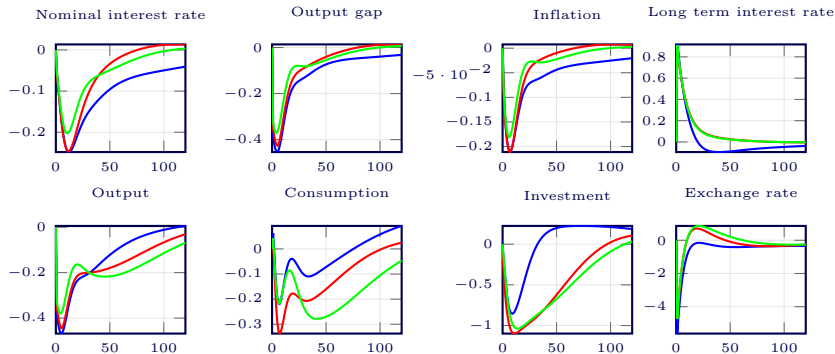
- Backward model with UIP,
- Full MCE,
- Hybrid mode (MCE in wage, wapro, financial and exchange rate blocks).

Responses to a one point shock on the nominal interest rate



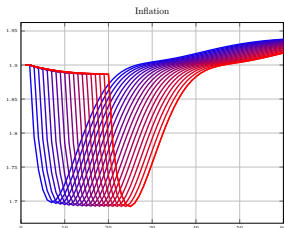
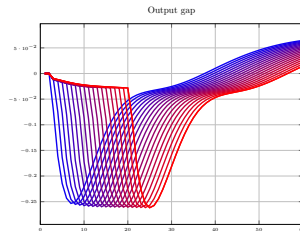
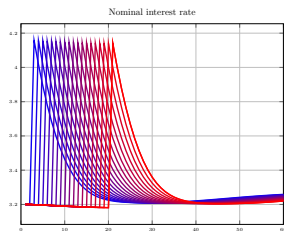
— Backward with UIP model, — Full MCE model, — Hybrid model

Responses to a one point shock on the term premium



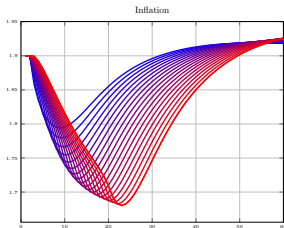
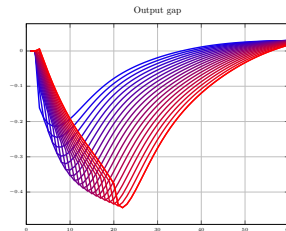
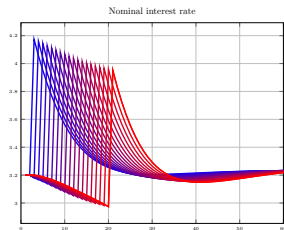
— Backward with UIP model, — Full MCE model, — Hybrid model

Forward guidance (Backward with UIP model)



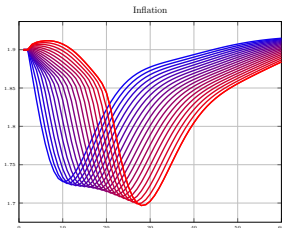
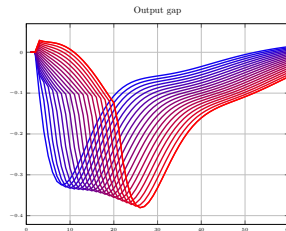
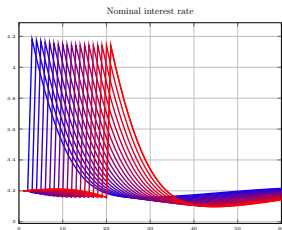
- t=1
- t=2
- t=3
- t=4
- t=5
- t=6
- t=7
- t=8
- t=9
- t=10
- t=11
- t=12
- t=13
- t=14
- t=15
- t=16
- t=17
- t=18
- t=19

Forward guidance (full MCE)



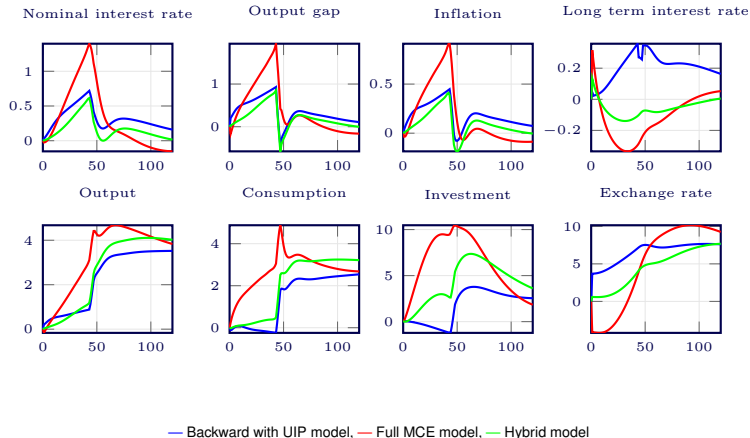
t=1 t=2
t=3 t=4
t=5 t=6
t=7 t=8
t=9 t=9
t=10 t=11
t=12 t=13
t=14 t=15
t=16 t=17
t=18 t=19

Forward guidance (Hybrid model)



t=1 t=2
t=3 t=4
t=5 t=6
t=7 t=8
t=9 t=9
t=10 t=11
t=12 t=13
t=14 t=15
t=16 t=17
t=18 t=19

Expected TFP growth shock in 10 years for one year ($4 \times 1\%$)



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Next steps

Main deliverables for the end-August milestone

- Freeze the benchmark backward model
- Systematic comparison of shock transmission between the backward model, the hybrid expectations model and the full MCE model
- Further examples of anticipated shocks
- Policy application

Follow-up activities

Possible workstreams

- Stationary version of the model and linear approximation
 - Tractability of stochastic simulations
 - Filtering
- Empirical validation of the MCE model
 - Indirect inference on the Wapro/wage blocks
 - System-wide inference using the linear model
- Policy analysis with the MCE model
 - Nesting a term-structure model into the ECB-BASE-MCE
 - Optimal policy projections