SMC with Dynare

stephane.adjemian@univ-lemans.fr

September, 2024

Introduction

- Estimating nonlinear models with particle-based likelihood approximations is difficult due to the nondifferentiability of the target (resampling).
- Estimation of linear models with Markov Chains can also prove to be tricky (Where is the mode? Multimodality?) or slow.
- Sequential Monte Carlo, see Herbst and Schorfheide (2014), can help $\rightarrow$ `hssmc` option.
- Importance sampling + Homotopy over distributions.

Importance sampling, I

- Let X be a random variable. We want to compute the expectation of $\varphi(X)$:

$$\mathbb{E}_p\left[\varphi(X)\right] = \int p(x)\varphi(x)\mathrm{d}x \xleftarrow[N\to\infty]{} \frac{1}{N}\sum_{i=1}^{N}\varphi(x_i)$$

- If we obtain $\{x_i\}_{i=1}^{N}$ from another distribution with density $q(x)$:

$$\mathbb{E}_p\left[\varphi(X)\right] = \int q(x)\frac{p(x)}{q(x)}\varphi(x)\mathrm{d}x = \int q(x)\tilde{\omega}(x)\varphi(x)\mathrm{d}x = \mathbb{E}_q\left[\tilde{\omega}(X)\varphi(X)\right]$$

  with $\tilde{\omega}(x)$ the (unnormalized) importance weight.

- We can approximate the expectation sampling from $q(x)$:

$$\frac{1}{N}\sum_{i=1}^{N}\omega(x_i)\varphi(x_i) \xrightarrow[N\to\infty]{} \mathbb{E}_p\left[\varphi(X)\right]$$

  with $\omega(x)$ the normalized importance weight.

Importance sampling, II

- IS works as long as the support of the targeted distribution (p) is included in the support of the instrumental distribution (q).

- How do we choose distribution $q(x)$?

- An optimal instrumental distribution q, minimizing the variance of the approximation of $\mathbb{E}_p[\varphi(X)]$, would depend on p and φ.

$\rightarrow$ Therorem 3.12 in Robert and Casella (2004)

- Is it possible to find a reasonable intrumental distribution for a posterior distribution? The prior?

- Probably better not to "jump" directly to the posterior distribution...

Sequential importance sampling, I

Target $p\left(\theta|\mathcal{Y}_T\right) \propto p(\theta)p(\mathcal{Y}_T|\theta)$

- Consider the "simplified" object $p\left(\theta\right)p\left(\mathcal{Y}_T|\theta\right)^{\phi}$
- $p\left(\theta\right)$ is a good instrumental distribution for $p\left(\theta\right)p\left(\mathcal{Y}_T|\theta\right)^{\phi}$ for small values of ϕ.
- Consider a sequence of $\{\phi_i\}_{i=1}^{M}$ with $\phi_i < \phi_j$ for all $j > i$ and $\phi_M = 1$.
- Use $p\left(\theta\right)$ as an intrumental distribution for $p\left(\theta\right)p\left(\mathcal{Y}_T|\theta\right)^{\phi_1}$ and $p\left(\theta\right)p\left(\mathcal{Y}_T|\theta\right)^{\phi_i}$ as an instrumental distribution for $p\left(\theta\right)p\left(\mathcal{Y}_T|\theta\right)^{\phi_{i+1}}$.
- Herbst and Schofheide consider $\phi_n = \left(\frac{n}{M}\right)^{\lambda}$ with $\lambda > 1$ (convexity).

Sequential importance sampling, II

- Suppose in step $m-1$ we have a particle swarm $\{\theta_i^{(m-1)}, \omega_i^{(m-1)}\}_{i=1}^N$.
- Then, in step $m = 1, \ldots, M$:
  - **Correction** Reweight the particles $\tilde{\omega}_i^{(m)} = \omega_i^{(m-1)} p\left(\theta_i^{(m-1)} | \mathcal{Y}_T\right)^{\phi_m - \phi_{m-1}}$ and normalize:

$$\omega_i^{(m)} = \frac{\tilde{\omega}_i^{(m)}}{\sum_{i=1}^N \tilde{\omega}_i^{(m)}}$$

  - **Resampling (optional)** $\rightarrow$ new particle swarm $\{\tilde{\theta}_i^{(m)}, \frac{1}{N}\}_{i=1}^N$
  - **Mutation** MH step(s) $\rightarrow$ updated particle swarm $\{\theta_i^{(m)}, \frac{1}{N}\}_{i=1}^N$

Sequential importance sampling, III

Options for the estimation command

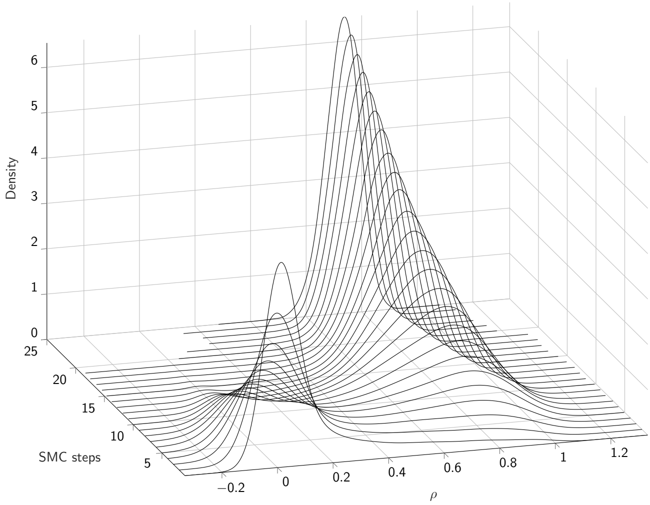
```
posterior_sampling_method = 'hssmc'
```

```
posterior_sampling_options = ('particles', 20000,
                              'steps', 25,
                              'lambda', 2,
                              'target', 0.25,
                              'scale', 0.5)
```

Sequential importance sampling, IV

#Iter.	lambda	ESS	Acceptance rate	scale	resample	seconds
1	0.0000	0.00000E+00	0.0000	0.000	no	10.68
2	0.0017	1.43623E+04	0.1933	0.525	no	6.21
3	0.0069	7.75007E+03	0.2281	0.514	yes	4.67
4	0.0156	1.43886E+04	0.2601	0.509	no	4.72
5	0.0278	8.03816E+03	0.2901	0.511	yes	4.71
6	0.0434	1.60505E+04	0.2994	0.519	no	4.79
7	0.0625	1.06019E+04	0.2967	0.529	no	4.70
8	0.0851	6.45163E+03	0.3125	0.539	yes	4.71
9	0.1111	1.73358E+04	0.3090	0.551	no	4.68
10	0.1406	1.25754E+04	0.3125	0.563	no	4.75
11	0.1736	8.36531E+03	0.3136	0.576	yes	4.82
12	0.2101	1.80047E+04	0.3252	0.590	no	4.76
13	0.2500	1.43417E+04	0.3295	0.606	no	4.77
14	0.2934	1.09151E+04	0.3367	0.623	no	4.76
15	0.3403	8.26027E+03	0.3257	0.641	yes	4.91
16	0.3906	1.89134E+04	0.3299	0.659	no	4.89
17	0.4444	1.67341E+04	0.3350	0.677	no	4.90
18	0.5017	1.43265E+04	0.3270	0.697	no	4.89
19	0.5625	1.20642E+04	0.3192	0.716	no	4.89
20	0.6267	1.00683E+04	0.3123	0.734	no	4.93
21	0.6944	8.38327E+03	0.2868	0.751	yes	5.05
22	0.7656	1.94409E+04	0.2863	0.762	no	5.03
23	0.8403	1.81259E+04	0.2848	0.773	no	5.09
24	0.9184	1.64313E+04	0.2836	0.783	no	4.99
25	1.0000	1.46032E+04	0.2787	0.794	no	5.03

Sequential importance sampling, V



Sequential importance sampling, VI

TODO

- More options (number of mutation steps, resampling algorithm, ...).
- Complete posterior computations (bayesian IRFs, forectasts, ...).
- No reason to start from the prior distribution.
- We could start from the (empirical) posterior distribution of another model (with the same set of estimated parameters).
- The formula for the correction would not be as nice.

References

Herbst, Edward and Frank Schorfheide (2014). "Sequential Monte Carlo Sampling For DSGE Models". In: *Journal of Applied Econometrics* 29(7), pp. 1073–1098.

Herbst, Edward and Frank Schorfheide (2016). *Bayesian Estimation of DSGE Models*. Princeton University Press.

Robert, Christian P. and George Casella (2004). *Monte Carlo statistical methods*. Springer Verlag.